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Several remarks on "Comments" by A. Moroz

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Abstract

We make a couple of remarks on "Comments" due to A. Moroz which were addressed to our recent letter [1].

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In this note we wish to make a couple of remarks on "Comments on 'Differential cross section for Aharonov–Bohm effect with non-standard boundary conditions' " (referred to as Comments in what follows) due to A. Moroz and addressed to our recent letter [1]. In fact, these remarks have been revised as a consequence of the revision of Comments. In the very beginning it should be emphasized that in our letter [1] we didn't pretend and attempt to do anything more than claimed in the introductory part, and this was to complete the results of the preceding paper [2] by doing some elementary numerical analysis, plotting a couple of graphs, and providing them with some basic discussion. However all the involved formulae were derived in [2] (of course, the rotational symmetry was mentioned there, too). The basic result of [2] consists in finding a general form of boundary conditions, depending on four parameters, imposed on a wave function in the case of idealized Aharonov–Bohm effect in the plane, and furthermore in derivation of a formula for the differential cross section with the corresponding Hamilton operator. The value of the magnetic flux is then another parameter, the fifth one, occurring in the problem.

It should be noted that approximately at the same time another paper appeared, [3], treating the same problem as we did in [2], using practically the same mathematical tools, and naturally arriving at the same results. So Comments by Moroz might have been better addressed to the papers [2] and [3] rather than to [1].

The mathematical machinery applied in [2] and [3] is based on the theory of self-adjoint extensions of symmetric operators which is nowadays quite a common tool suited for this type of problems. For example, just for the purpose of illustration, let us mention that in another relatively recent paper, [4], in its nature a similar problem, concerning this time a magnetic monopole in three-dimensional space, has been solved by the same method.

As I understood from Comments, Moroz had applied in his analysis a completely different technique based on a limit procedure when sending the radius of the flux tube to zero. There is no doubt that this approach is highly interesting, too. For example one may hope to give this way the involved parameters a more concrete interpretation. Unfortunately I am not able to compare the results directly since I didn't deduce from Comments what was the precise definition of Hamiltonian, particularly what kind of boundary conditions were imposed on wave functions. Nevertheless some aspects seem to be clear. First of all, Comments are apparently concerned with the case when the s and p-wave are decoupled. Then the corresponding family of Hamiltonians depends only on two parameters, called Δ_{-n} and Δ_{-n-1} in formula (3) of Comments, while the complete solution admits coupling of the s and p-wave via boundary conditions, and consequently depends, as already mentioned, on four parameters.

As far as the rotational symmetry is concerned, this notion is interpreted somewhat differently in [2, 3] on one side and in Comments on the other side. This was not our aim to study symmetries of the differential cross section itself, for example with respect to the reflection $\varphi \to -\varphi$. A basically more essential observation has been made in [2, 3] (and graphically illustrated in [1]). The point is that the angular momentum need not be conserved, or, in other words, the angular momentum need not commute with the Hamiltonian. This effect happens provided the s and p-wave are coupled, and this is what we called the violation of the rotational symmetry. For the differential cross section this means that it depends non-trivially on both the scattering and incident angle, φ and φ_0, and not merely on their difference $\varphi - \varphi_0$. Note that only the scattering angle φ occurs in formula (3) of Comments, with φ_0 being set to 0.

Furthermore, let us make a short remark on the number of bounded states. The situation is slightly more complicated than mentioned in Comments. The possible number of bounded states is 0, 1 or 2. This means that even with non-standard boundary conditions it may happen that there are no bounded states. The dependence of the number of bounded states on the choice of boundary conditions has been analyzed quite explicitly both in [2] and [3].

As for the longer history, the fact that the two critical sectors of angular momentum admit more general boundary conditions than the regular one has been known for a long time, see for example the now classical paper [5], though this possibility was not exploited systematically until recently. We have to admit that we missed the work [6]. No doubt it should be included among the references of [1]. On the other hand I'd like to point out that the case with the s and p-wave decoupled was studied in several works prior to the paper [7] referred to in Comments (see [8, 9]).

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